Mortality models and longevity risk

Henk van Broekhoven, 30 June 2010

Presentation

- **Best Estimate mortality**
  - **Alternative smoothing method**
  - **Modelling high ages**
  - **Trend calculations**
  - **Working with a goal table**
- **Uncertainty trend**

Definition Life Risk

- **Life Risk relates to the deviations in timing and amount of cash flows due to incidence or non-incidence of death**
  - **Deviations relative to the Best Estimate Assumptions**
- **Overall mortality can be described by a probability function in which the Expected Value is the Best Estimate.**

Best Estimate Mortality

- **Current Estimate Mortality rates can be analysed into two parts:**
  - **Level**
  - **Trend**
- **The distribution is defined by the following sub-risks:**
  - **Volatility**
  - **Uncertainty Trend**
  - **Uncertainty Level**
  - **Extreme event risk (Calamity)**

Best Estimate Mortality

- Level
  - Mortality insured population not the same as population mortality. This difference also depends on period since issue
  - In some countries special mortality tables, that can vary by product (industry tables)
  - Still own company's observations can be different

Best Estimate Mortality

- **Level**
  - **Often age adjustments are used to reflect the difference insured-whole population**
  - **Age adjustments are by nature inexact**
  - **Better (age-dependent) factors**

Best Estimate Mortality

- **Trend**
- **Change in mortality/Life expectancy over time in history because of:**
  - **Medical developments (+)**
  - **Environment (+ or -)**
  - **Behaviour (+ or -)**
  - **New diseases (-)**

Best Estimate Mortality

- **Trend**
  - **Development over time not constant**
  - **Even periods with increasing rates (1951-1975 for male age 45-75)**

Best Estimate Mortality

- Trend possible models:
  - By cause of death
    - Problem new causes of death
  - By structure
    - Child – accident hump – constant part – aging part
  - General –independent of causes of death
  - Expert opinion

  - --- combinations---

Best Estimate Mortality

- **Trend, some remarks**
  - **Always better to use whole population mortality, not too heavily smoothed**
  - **Compare own results with other results, like in surrounding countries**
  - **Result should look reasonable**
  - **How far can we look into the future??**

Best Estimate Mortality

- Not enough data available to develop a good trend analysis

- Use information from surrounding countries or comparable countries
  - Possible added to available own data

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Best Estimate Mortality Trend

- **Use a general model**
  - **For example: Lee Carter, P-Spline, continuation trend by age**
- **Use population mortality (not insured population)**
- **Smoothed, but not using Gompertz or Makeham models**
  - **Alternative "*Van Broekhoven Algorithm*"**

The Van Broekhoven Algoritme

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New smoothing system

- **The vB algorithm**

- **Moving average weighted by a quadratic function through a transformation of the qx**

Transformation

$$f(x) = \ln - \ln\{1 - q(x)\}$$

Estimation

$$f(x) = a + bx + cx^2$$

Algorithm

- **To find the a,b,and c for a certain x use the observations for**

  x-5,…, x-1, x, x+1,…, x+5

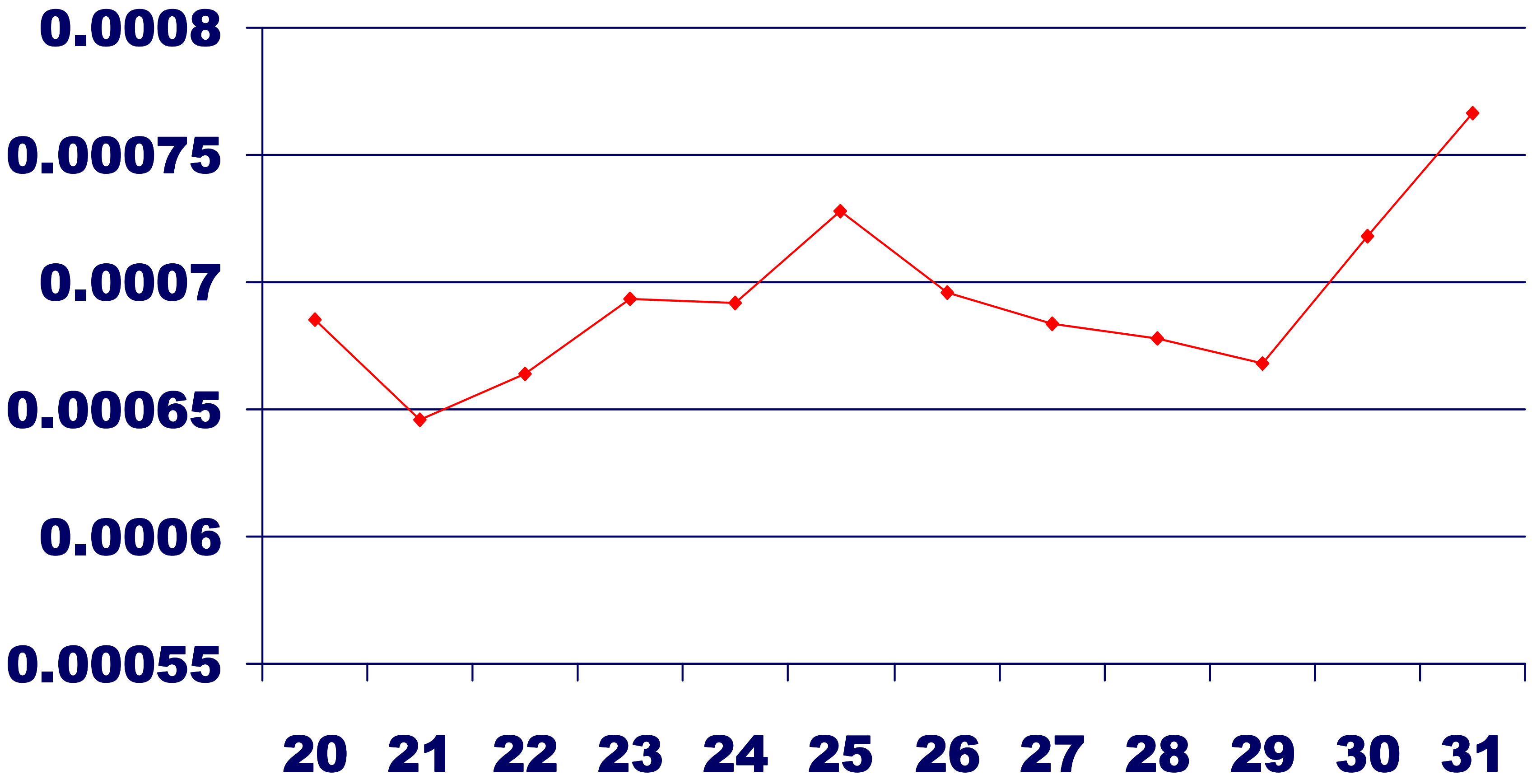
  - **so 11 observations (observations at x+0.5, use 10 observations)**
  - **using least squares**
- **and do that for each x**

Algorithm

- **The smoothed qx follows:**

$$q(x) = 1 - e^{-e^{\hat{f}(x)}}$$

The Algorithm movie



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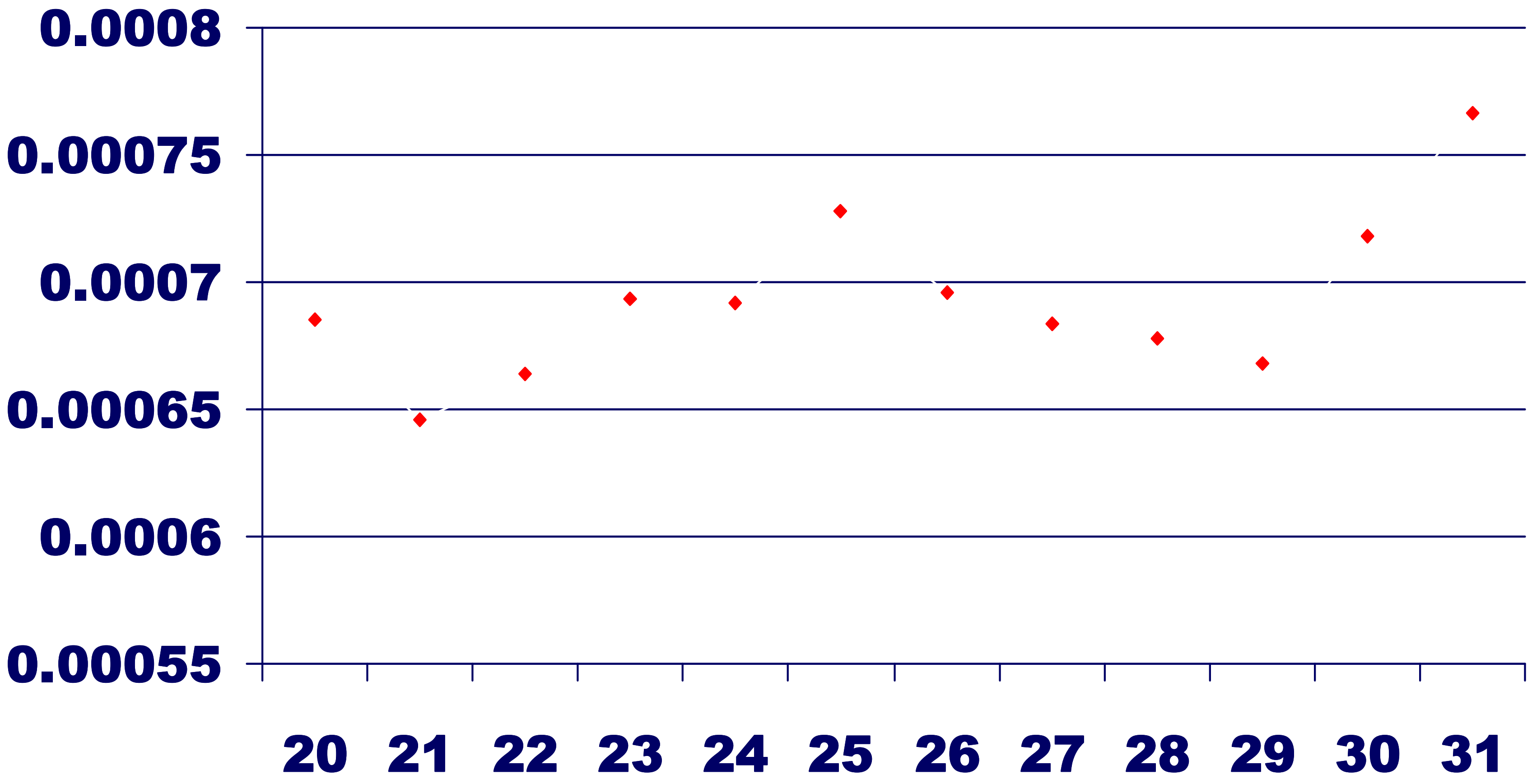
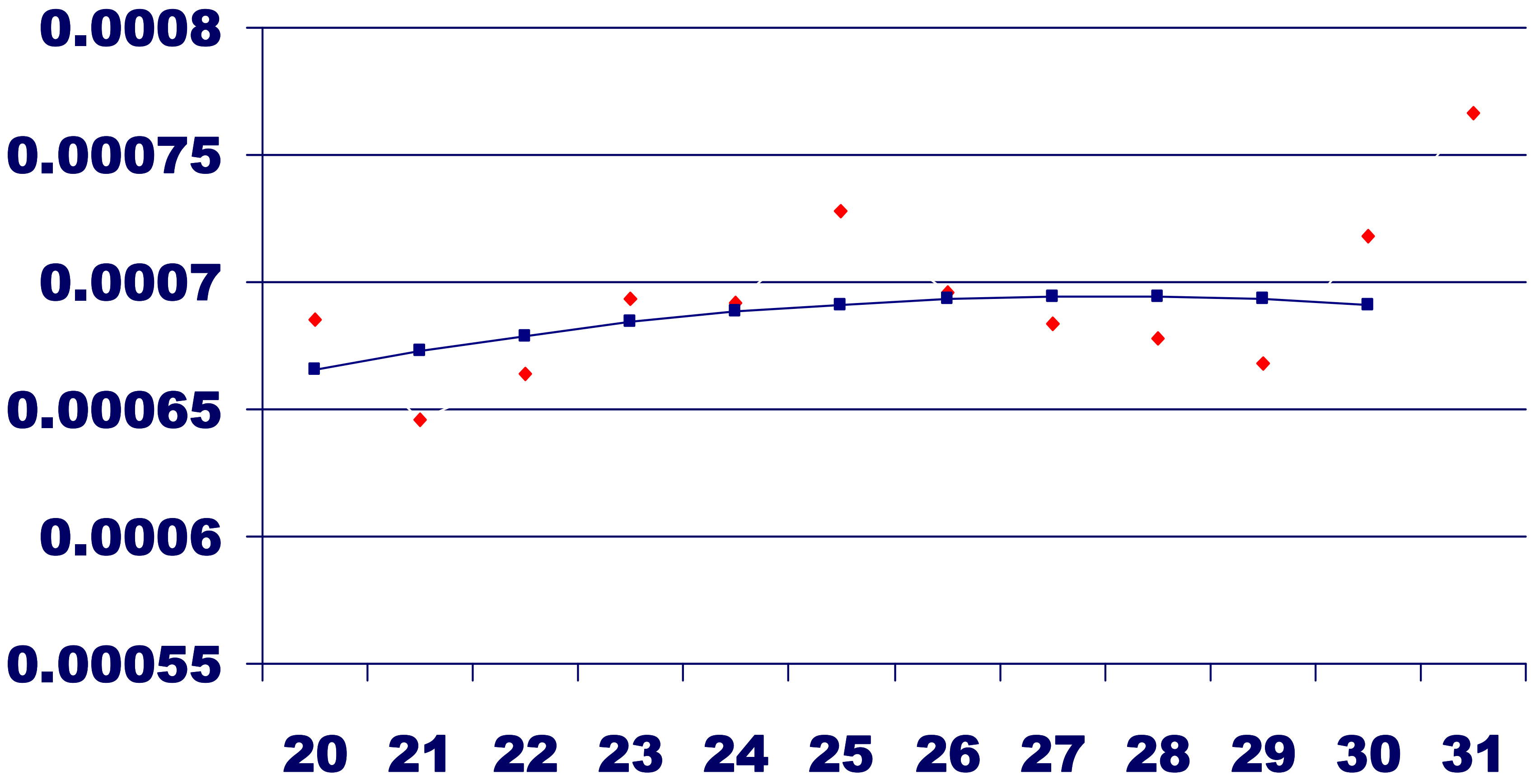
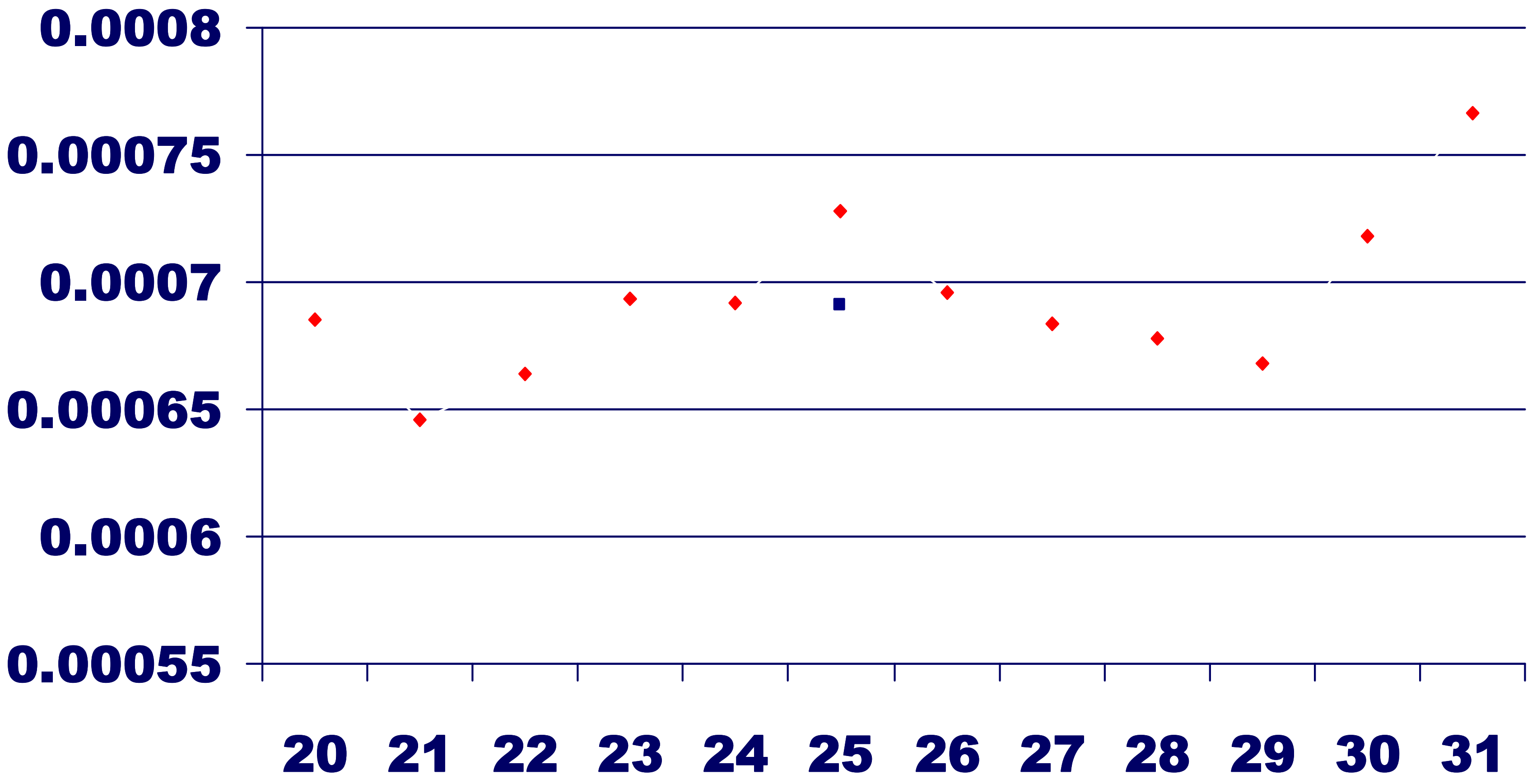
The Algorithm movie



The Algorithm movie



The Algorithm movie



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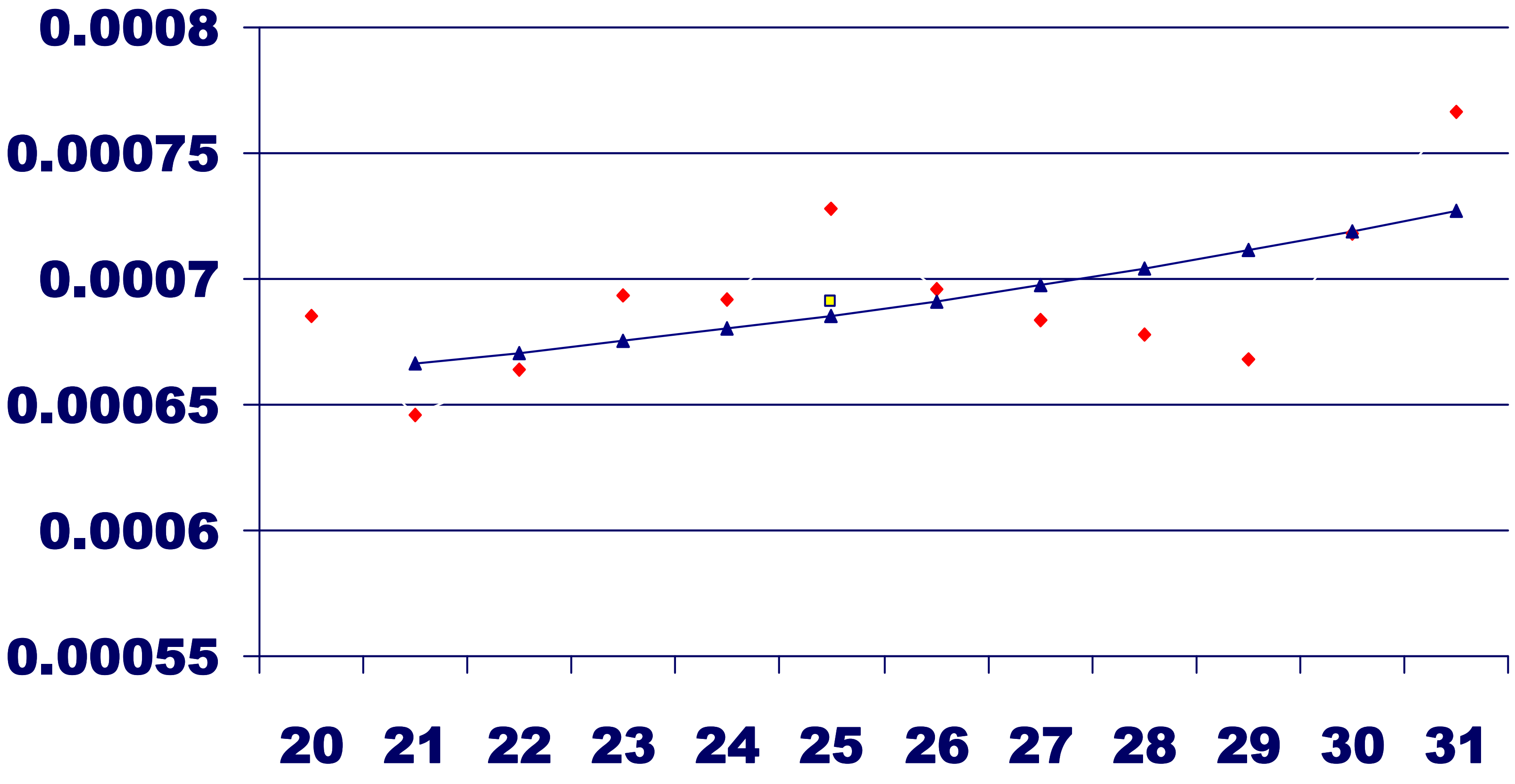

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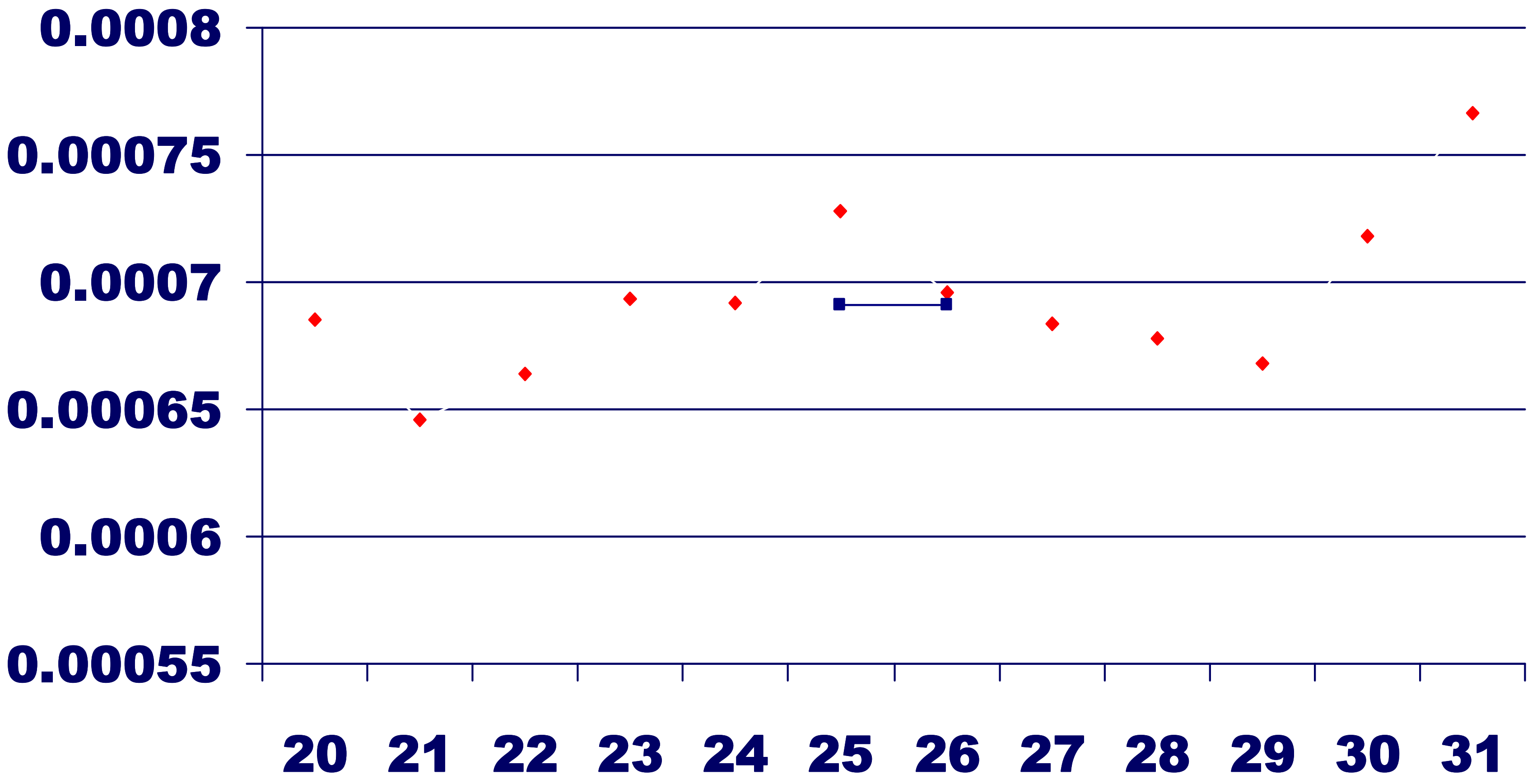

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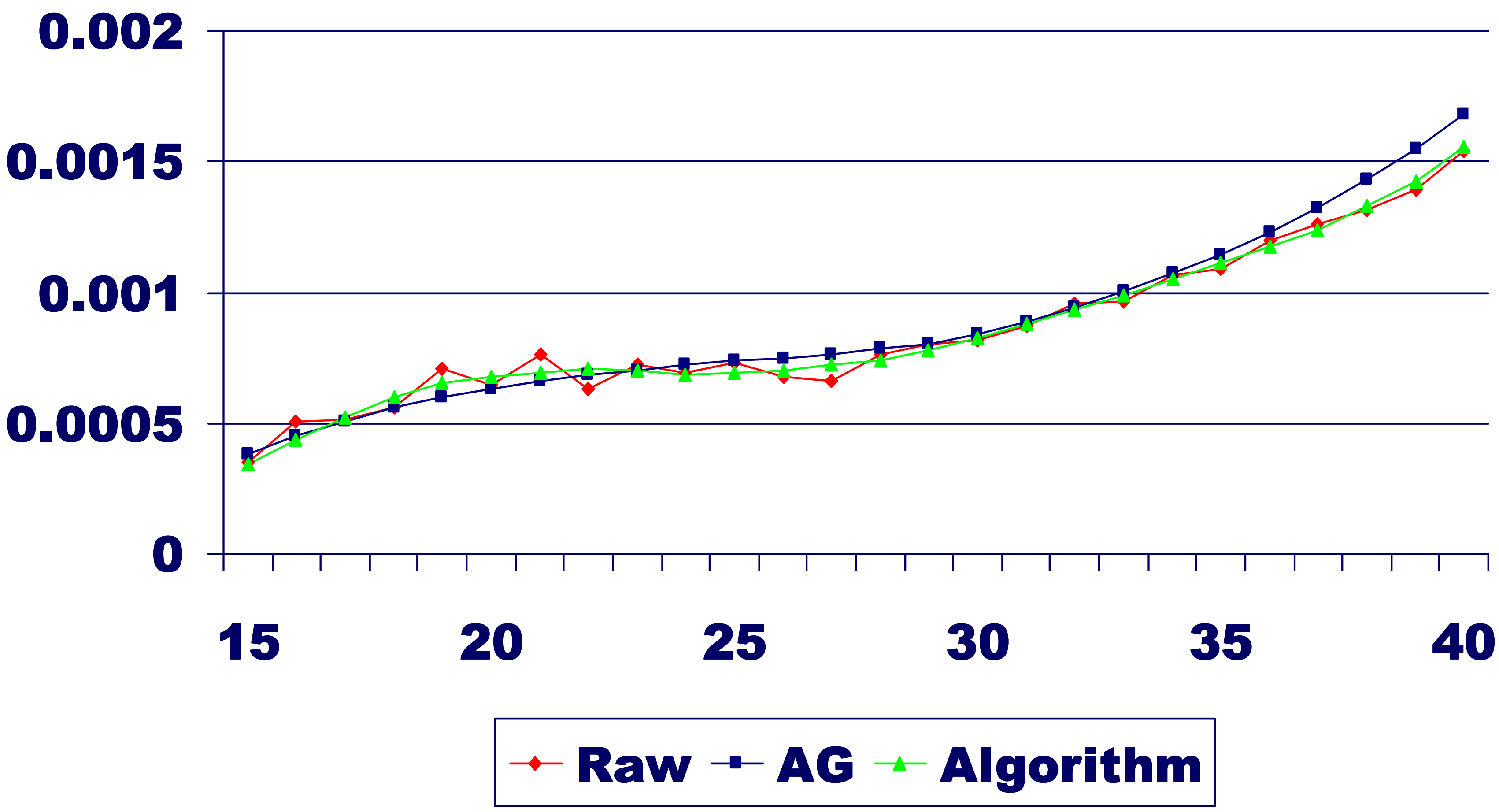
Method tested in 1990-1995 table



Some results for the life expectancy

Including high ages model

Age	Male		Female	
	raw	smoothed	raw	smoothed
0	75.055	75.059	80.479	80.481
25	51.021	51.023	56.178	56.181
45	31.890	31.888	36.836	36.837
65	14.981	14.983	19.125	19.124
85	4.635	4.625	5.836	5.833

Modelling the very high ages

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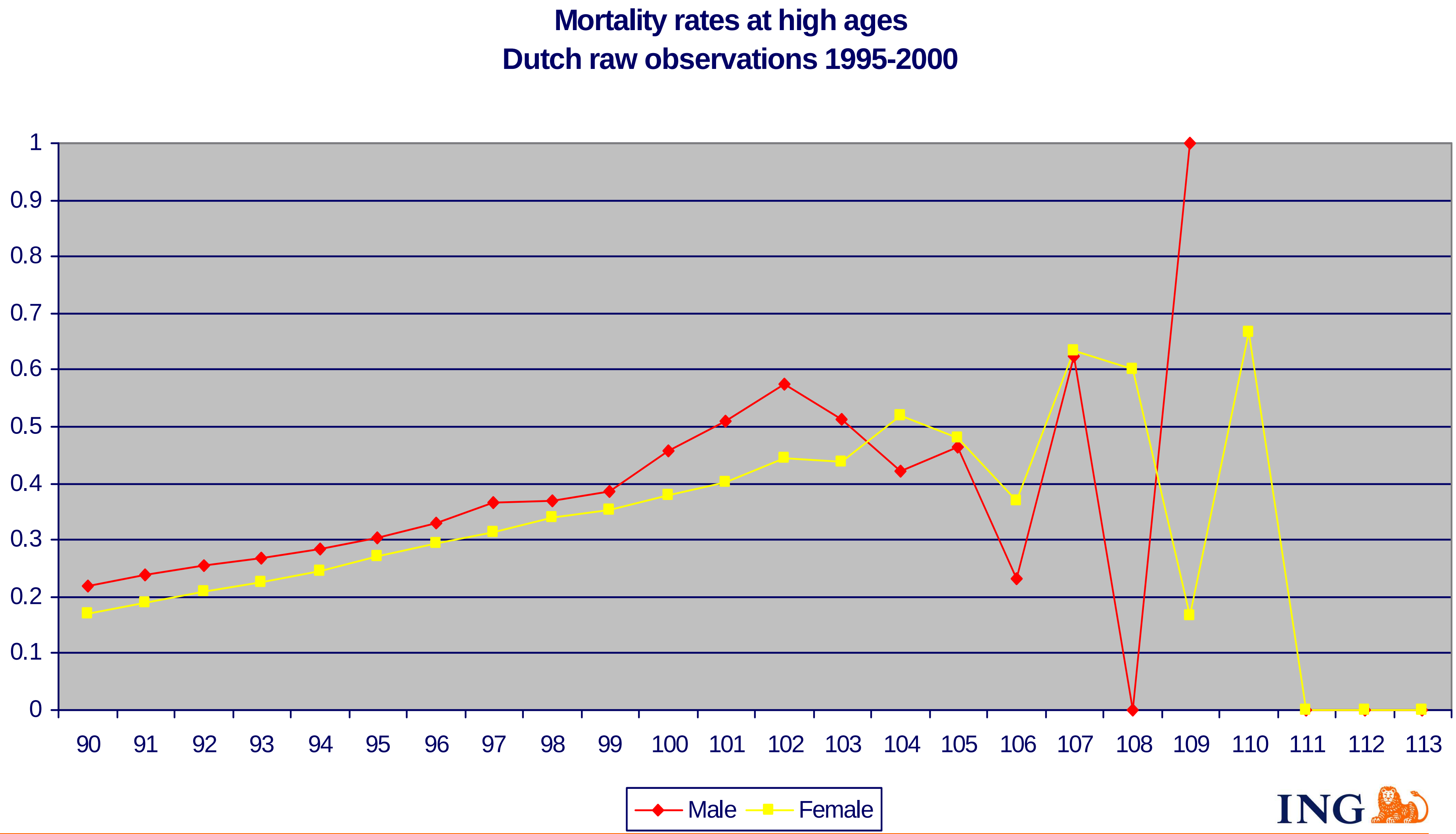
Modelling the high ages



Modelling the high ages

- **The Van Broekhoven Algorithm doesn't work at the high ages:**
  - **Transformation doesn't work if:**
  - $$q(x) = 0 \quad or \quad 1$$
  - **Observations too volatile**

Life expectancy age 100

Period	Male	Female
1971-1975	1.78	1.92
1976-1980	1.91	2.18
1981-1985	1.96	2.09
1986-1990	1.96	2.12
1991-1995	1.79	1.92
1996-2000	1.55	1.93

Modelling the high ages

Conclusion:

- No increase over time of the life expectancy
  - Perhaps even a decrease
- Hardly difference between male and female

A model for the high ages

- **Following the Gompertz law from a certain age x0:**

$$fn(x) = -\log\{1 - q(x)\}$$

$$fn(x+1) = fn(x) \times \alpha$$

$$so \quad fn(x_0 + t) = fn(x_0) \times \alpha^t$$

A model for the high ages

- **This can be translated into:**

$$q_x' = 1 - e^{e^{\alpha(x-x_0)} \ln p_{x_0}'}$$

With: p_x' **is the 1 year survival chance based on the smoothed (algorithm) table at age x**

A model for the high ages

- **We can also define the life expectancy derived from this formula**:

$$\bar{e}_{x_0} = 0.5 + \sum_{x_t = x_0} \prod_{x = x_0}^{x_t} e^{e^{\alpha(x - x_0)} \ln p'_{x_0}}$$

With $\bar{e}_x$ Is life expectancy based on the raw table

A model for the high ages

$$\bar{e}_{x_0} = 0.5 + \sum_{x_t = x_0} \prod_{x = x_0}^{x_t} e^{e^{\alpha(x - x_0)} \ln p'_{x_0}}$$

Known at some x0

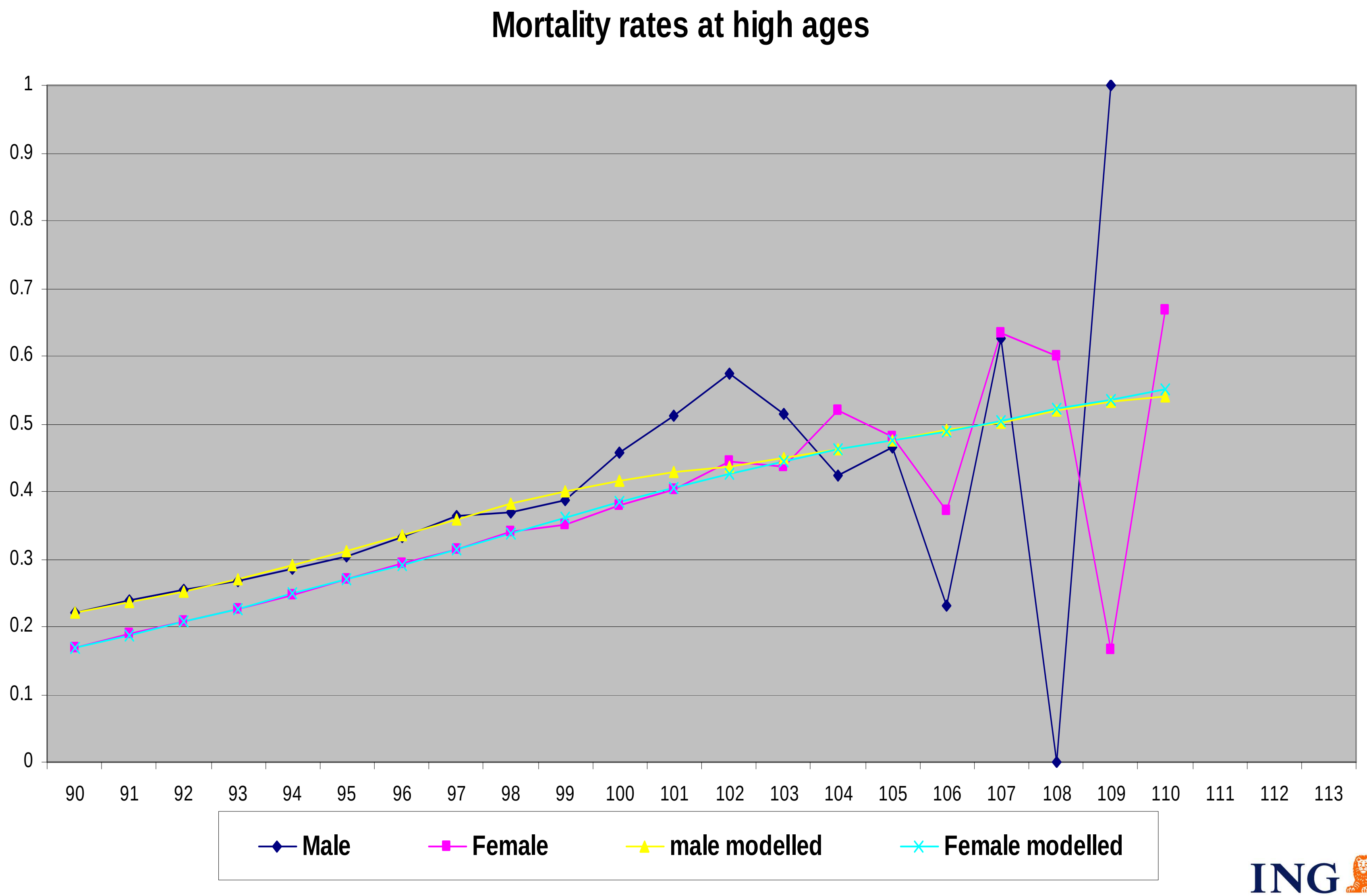
To be estimated (solver)

A model for the high ages

- **The x0 can be derived by calculating the tables for each x0 $\geq$ 90 possible and find, based on least square differences, the most optimal table**

- **Check the result on reasonability**

Results



Mortality rates at high ages
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113
Male
Female
male modelled
Female modelled

Many judgement decisions are involved in forecasting mortality

Judgement is required at every stage (Alders & De Beer, 2005)

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Best Estimate Mortality Trend

- **Compare with:**
  - **Other models**
  - **Surrounding countries**
- **Check for reasonability**

Best Estimate Mortality Trend

- **Continuation of recent trend, measured since last significant change in trend**
- **Trend factor:**

$$f(x) = \sqrt[p]{\frac{q(x;t)}{q(x;t-p)}}$$

Best Estimate Mortality Trend

- **Future mortality follows:**

$$q_{pop}(x; j+1) = q_{pop}(x; j) \times f(x)$$

- **So**

$$q_{pop}(x; j+t) = q_{pop}(x; j) \times f(x)^{t}$$

Other models

- The Lee Carter model (1992) is very popular in several countries and institutes.
- Stochastic model
- Some shortcomings, therefore a lot of variants made since 1992.

- The following sheets shows the differences with the simple trend model

Lee Carter vs simple trend model

Lee Carter (1992)

- The two factor Lee-Carter looks like:

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$$

- $\varepsilon_{x,t}$ → residual
- k_t → Overall level of mortality in year t
- b_x → Response at age x to change in overall mortality over time
- a_x → Average (over time) of log-mortality at age x
- $m_{x,t}$ → Central mortality rate at age x in year t

Lee Carter vs simple trend model

Lee Carter (1992)

- The two factor Lee-Carter looks like:

$$\log(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$$

- By minimising $\sum_t \sum_x \varepsilon_{x,t}^2$ using Singular-value decomposition (SVD) b(x) and k(t) are found
  - The over-time similarity in the age pattern is maximally utilised
  - 94% of age specific mortality change over time is accounted by change in k(t)

Lee Carter vs simple trend model

- Second step:
  - Adjust k(t) to fit 100% the reported lfe expectancies a time t (or # deaths)
  - Than:

$$k(t) = k(t-1) + c + e(t)\sigma$$

drift

Random fluctuations

  - [k(t)-k(t-1)]: independently identically distributed with mean c and standard deviation: σ

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Lee Carter vs simple trend model

$$c = \frac{1}{T}\sum_{t=1}^{T}[k(t) - k(t-1)] = \frac{k(T) - k(0)}{T}$$

Standard error e(t)σ :

$$se(e) = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}[k(t) - k(t-1) - c]^2}$$

Error in c:

$$se(c) = \sqrt{\frac{\sigma^2}{T}} \approx \frac{se(e)}{\sqrt{T}}$$

Lee Carter vs simple trend model

- Future given c, se(e) and set of samples for: e(T), e(s) {s>T} a trajectory of k(t), t>T is:

$$k(t) = k(T) + [c + se(c).e(T)](t - T) + se(e)\sum_{s=T+1}^{t} e(s)$$

- And:

$$\log[m(x,t)] = \log[m(x,T) + b(x)[k(t) - k(T)]]$$

Lee Carter vs simple trend model

The "Simple Model"

- **Trend factor:**

$$f(x) = \sqrt[p]{\frac{q(x;t)}{q(x;t-p)}}$$

Future mortality follows:

$$q_{pop}(x;j+1) = q_{pop}(x;j) \times f(x)$$

So $$q_{pop}(x;j+t) = q_{pop}(x;j) \times f(x)^t$$

Li Lee method

- In 2002 Li, Lee and Tuljapurda presented a research paper:

“*Using the Lee Carter Method to Forecast Mortality for Populations with Limited Data*”

Li Lee method

- With limited number of observations it is difficult to set the se(e) and se(c). We should know the fluctuations between the observation points (e.g. calendar year u(t))
- Still c can be estimated

$$c = \frac{k(T) - k(0)}{u(T) - u(0)}$$

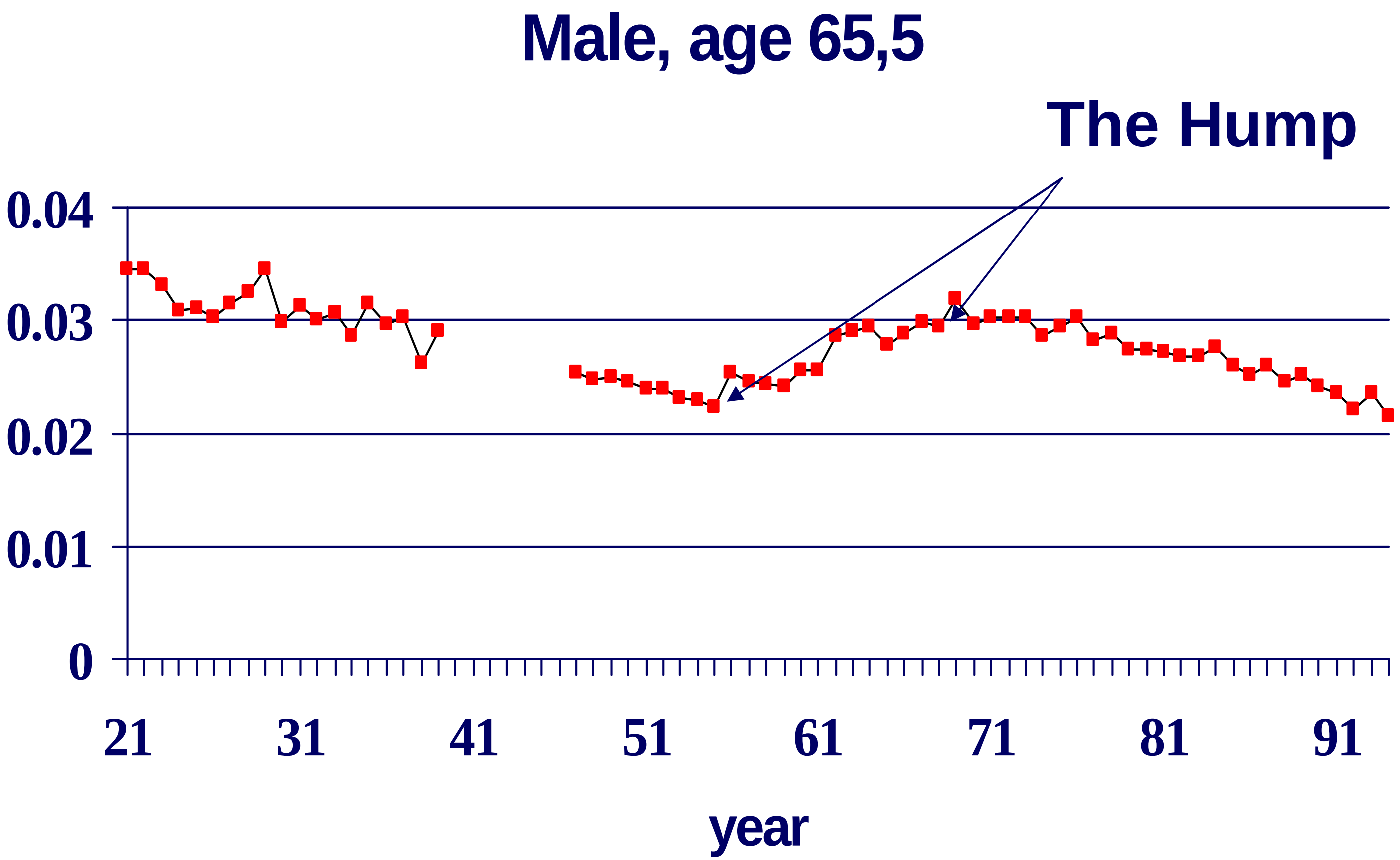
Li Lee method

- In the paper "Using Lee Carter Method to Forecast Mortality for Populations with Limited Data" the authors suggest to use the fluctuations from other countries, for example out of a G7 study by Tuljapurkar.
  - Conservatism can be included by using higher values than average.

Best Estimate Mortality Trend

- **Don’t be just a statistical**
- **Future table should be a mortality table**
- **Check with experts**
- **Try to explain strange observations**
    - **Example “The Hump”**

Mortality rate



Explaining the hump

- Only observed for male age 45-75 in the period 1951-1975
- Causes of death:
  - (car) accidents
  - (lung) cancer
  - heart diseases
- All reduced in the seventies:
  - medical development
  - behaviour

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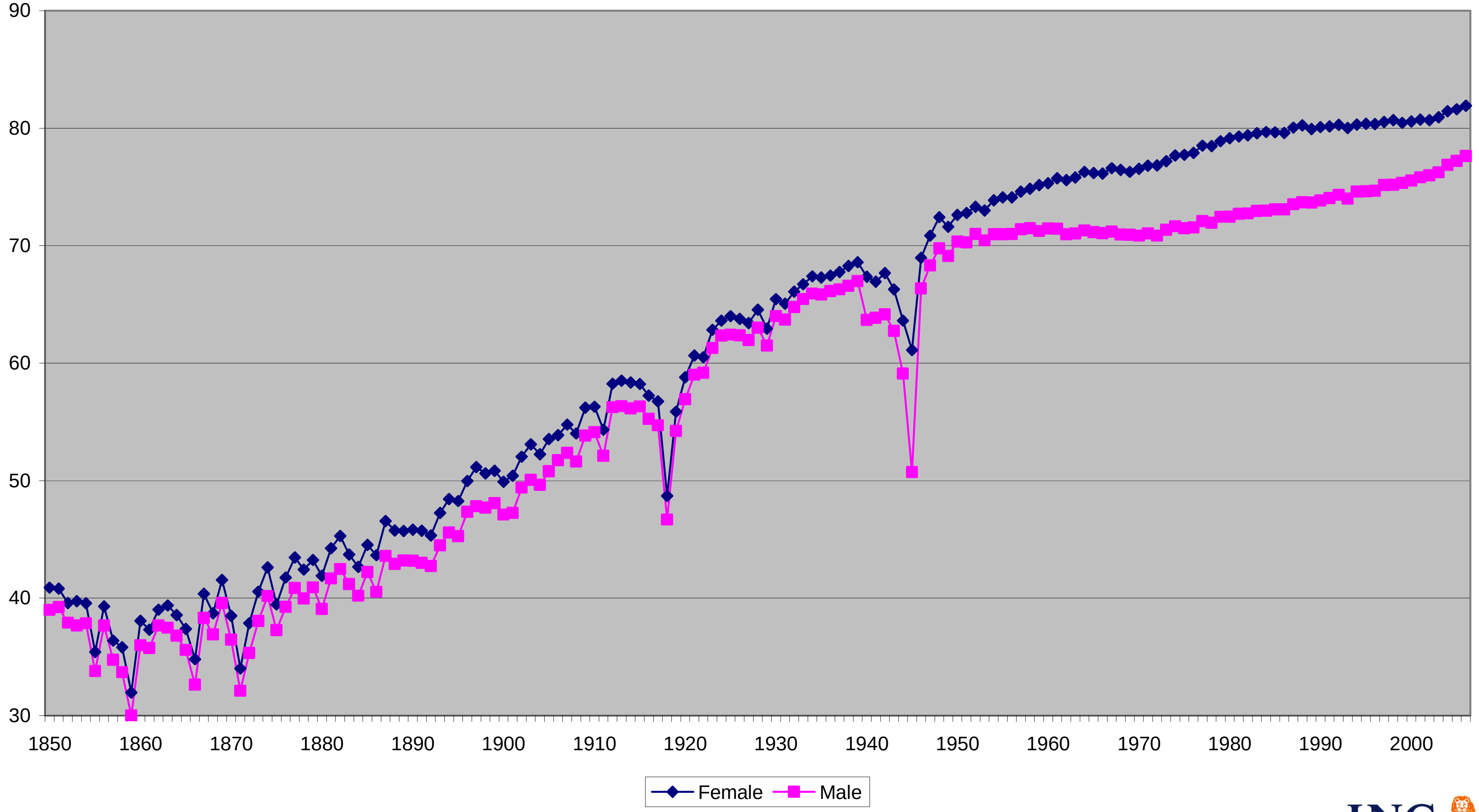
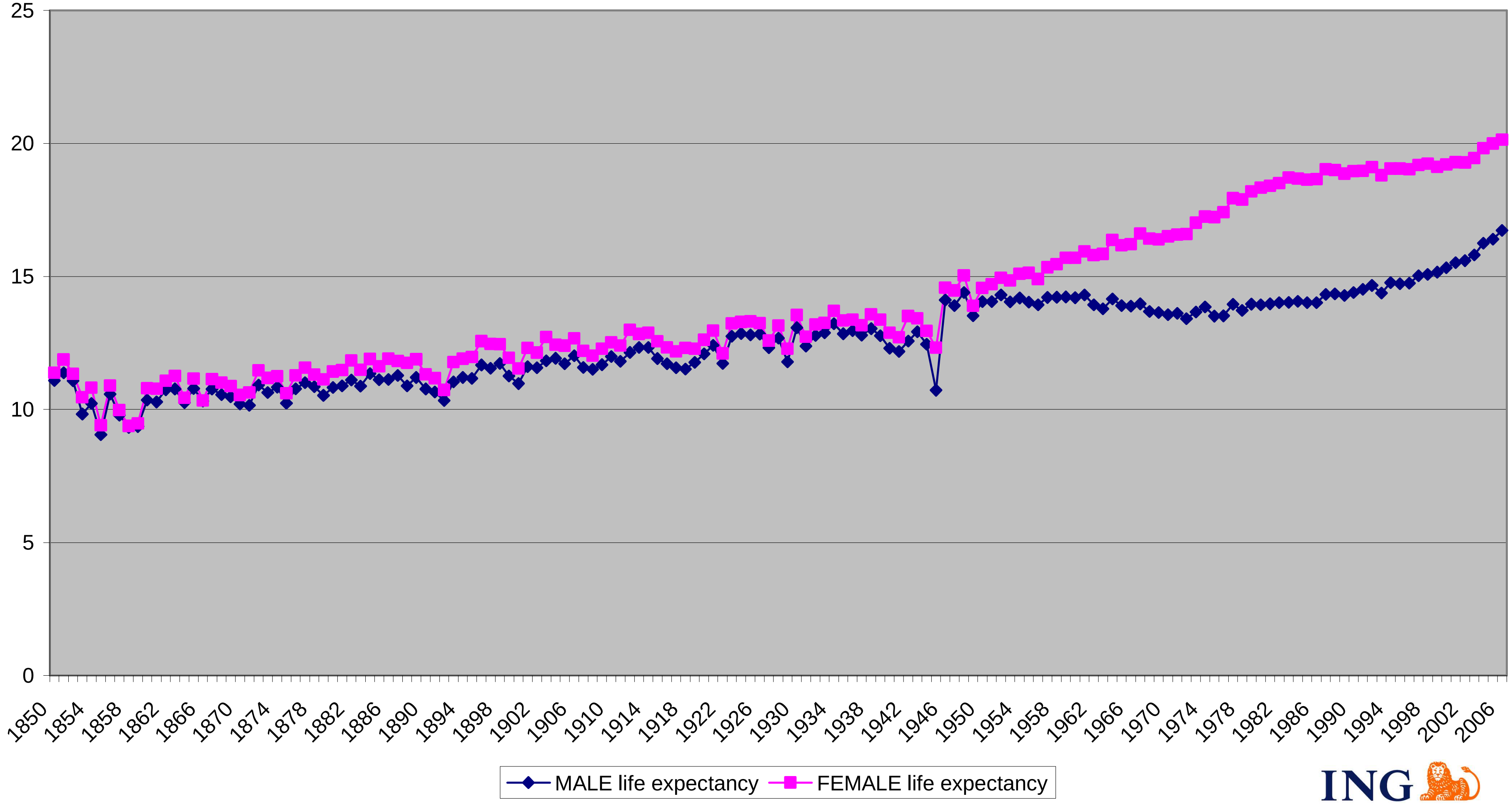
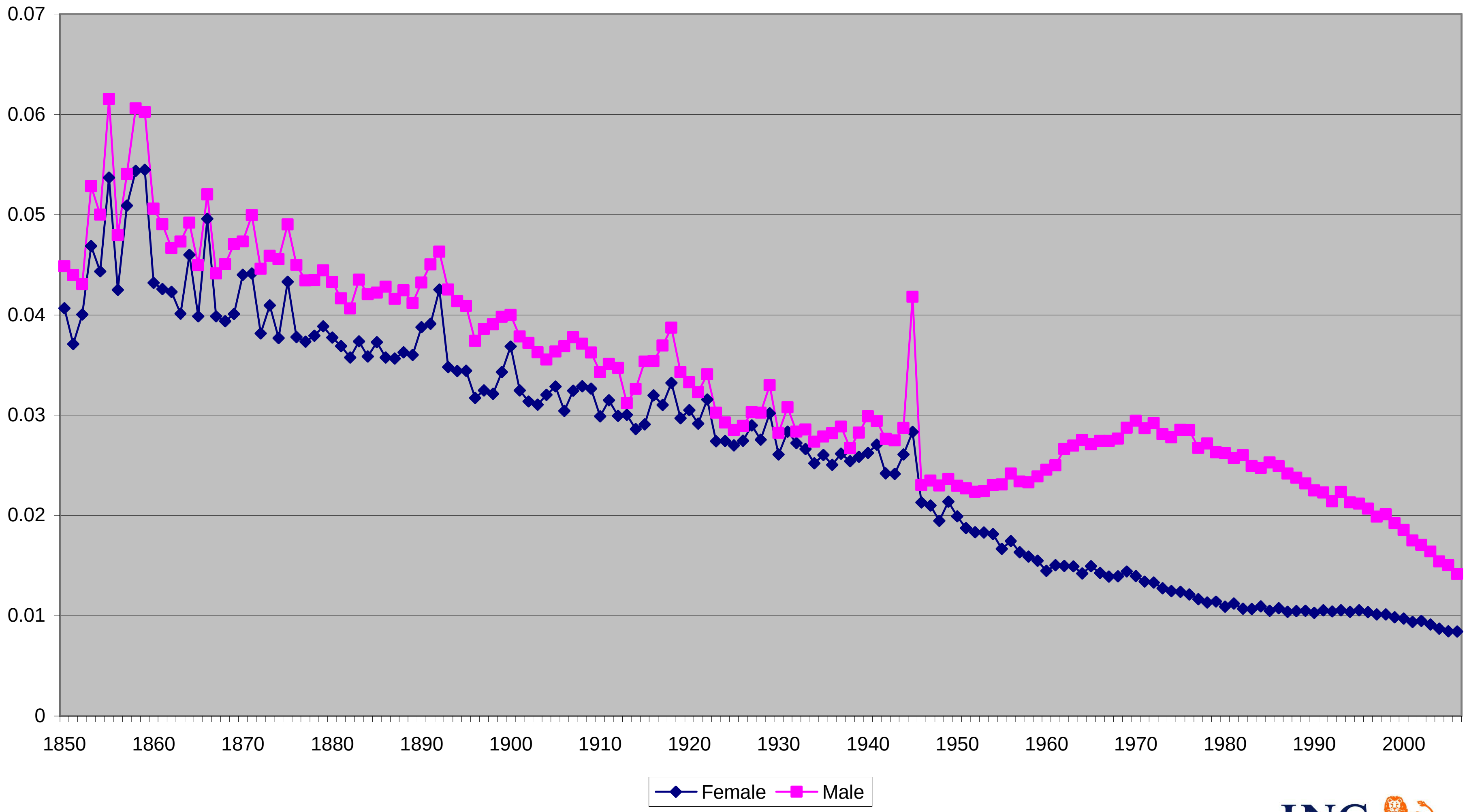
Life expectancy Netherlands age 0



history life expectancy age 65
Netherlands



Mortality rates age 65 Netherlands



Growing to a goal table

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Growing to a goal table

- **Producing a mortality prognosis over a very long period is difficult, even impossible.**
- **Therefore it is useful to look at other prognosis, like from medical experts, demographic persons etc.**
- **Sometimes it is not possible to produce a reliable trend, because of a lack of data.**

- **Therefore it can be useful not to extrapolate a local trend for a very long period, but grow to some future goal table.**
- **This is also necessary for comparing your own prognoses with other countries (they should stay close, for example by growing to a kind of average table by a region like Europe)**

Growing to a goal table

- **Normally (in standard model)**

$$q(x;j+t)=q(x;j)\times f(x)^t$$

- **Making the f(x) time dependent (f(x;t)):**

$$q(x;j+t)=q(x;j)\times\prod_{i=1}^{t} f(x;j+i)$$

Growing to a goal table

- **Define: f(x;j) is the local trend and**

$$f(x;j+i) = f(x;j) \times e^{i \times \alpha(x)}$$

- **So**

$$q(x;j+t) = q(x;j) \times \prod_{i=1}^{t} f(x;j) \times e^{i\alpha(x)}$$

- **And:**

$$q(x;j+t) = q(x;j) \times f(x;j)^{t} \times e^{\frac{\alpha(x)t(t+1)}{2}}$$

Growing to a goal table

- **Explaining:**

$$q(x; j+t) = q(x; j) \times f(x; j)^t \times e^{\frac{\alpha(x)t(t+1)}{2}}$$

When t increases the third factor will dominate the middle factor

So the local tend (f(x;j)) will become less important
as the time goes on

Growing to a goal table

- **Solving α(x) <j+t_g is the "goal year">:**

$$\alpha(x) = \frac{\log q(x; j+t_g) - \log q(x; j) - t_g \times \log f(x; j)}{\frac{1}{2} \times t_g \times (t_g + 1)}$$

Growing to a goal table

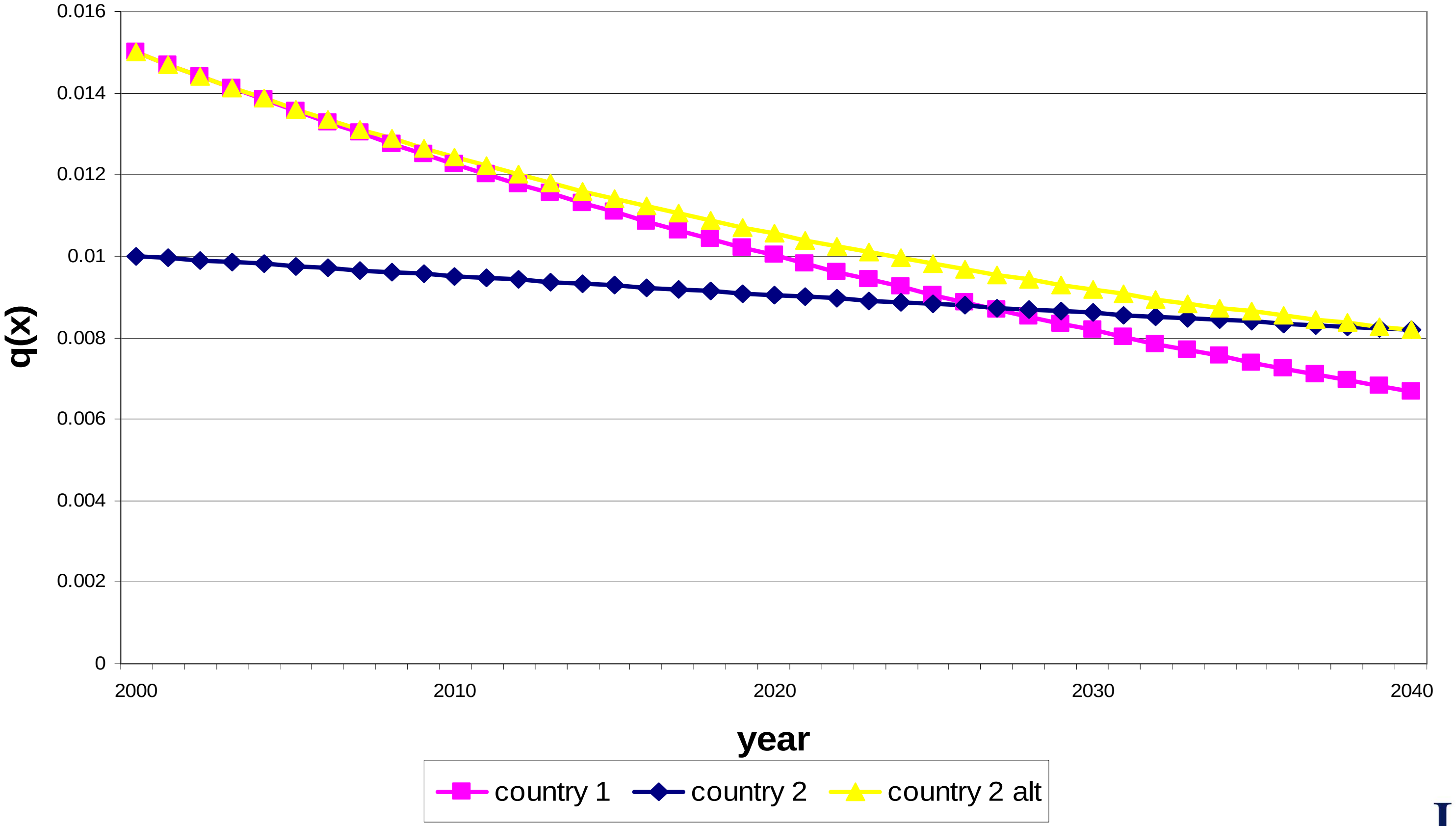
- **Extra conditions (not necessary):**

$$q(x;j) < q(x;j+t) \longrightarrow$$
$$j < p < j+t:$$
$$q(x;p) = q(x;j)$$

$$q(x;j) > q(x;j+t) \wedge q(x;j+p) < q(x;j+t) \longrightarrow$$
$$j+p < r < j+t:$$
$$q(x;r) = q(x;j+p)$$

Result

- At the start following the local trend
- At the end reaching “the goal”



Goal table

- The method of the goal table can be used in case:
  - Not enough information is available to create long term prognosis
  - The trend in one country is very steep compared to the surrounding (comparable) countries
  - The local trend is not in line with “normal” and therefore not applicable to predict the far future
    - Results in short term / long term model

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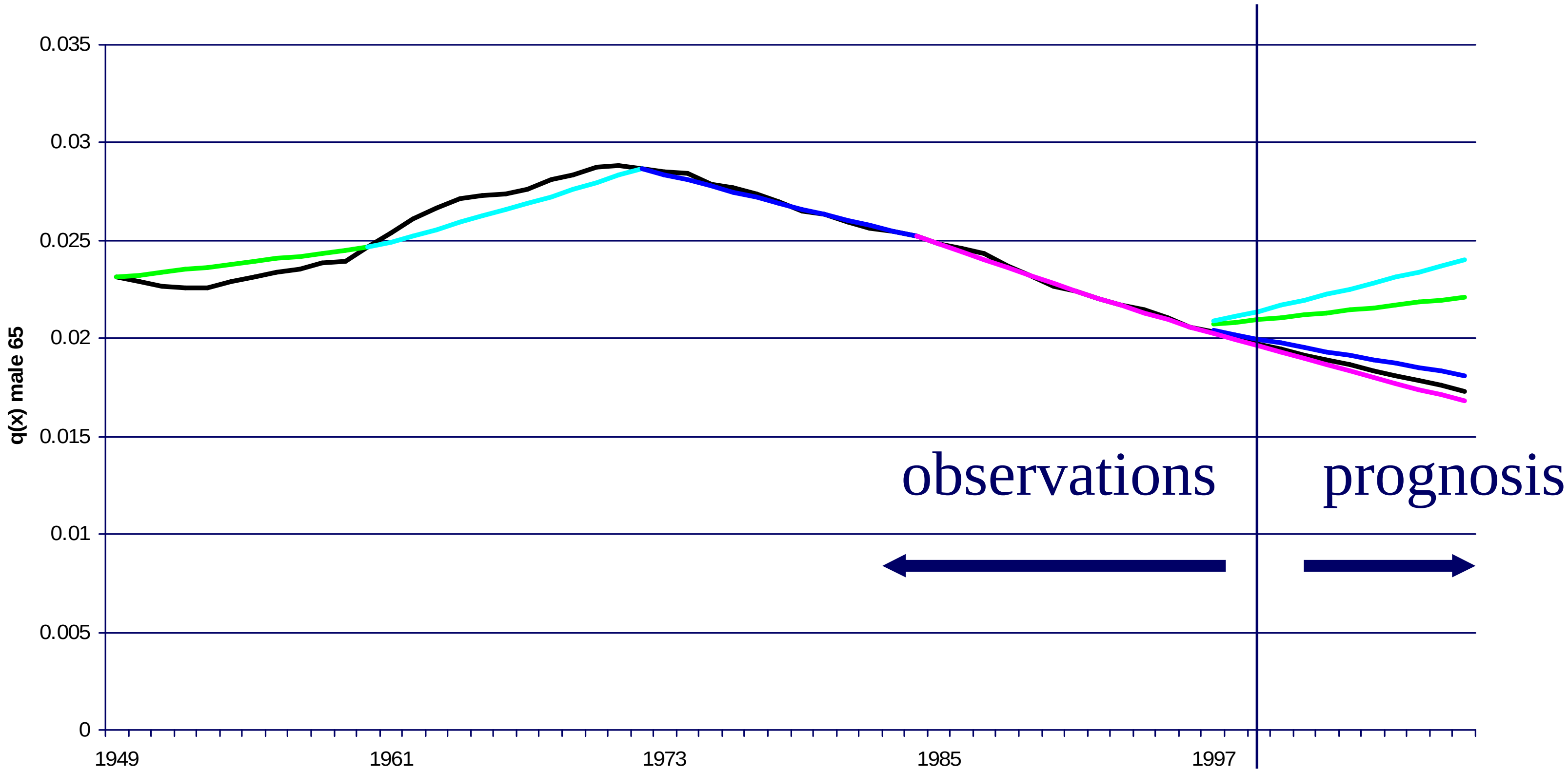
Uncertainty Trend

- It is impossible to predict a future trend.
- Medical development, enveronment, new diseases and resistance agains a medical cure can change trend (drift).
- Also volatility in the observations will cause uncertainty (random walk).

- We must try to say something about the uncertainty.

History Shows:
Several Trends Possible



Uncertainty Trend

- **The trend duration we use depends on the remaining average duration in the portfolio**

- **Duration short means less drift, more random walk**
- **Duration long means more drift, less random walk**

Uncertainty Trend

- **The yearly development factor f(x) can be calculated with other observed trends:**

  i = start period trend i observation and i+p end period trend i observation

$$f_i(x) = \sqrt[p]{\frac{q(x;i+p)}{q(x;i)}}$$

Uncertainty Trend

- **With each trend i the generation table i can be calculated**

$$q_i(x; j+t) = f_i(x)^t \times q(x; j)$$

Uncertainty Trend

- **With each generation table i: $q_i(x;t)$ liabilities are calculated: $liab_i$**
- **With each possible trend we get a liability.**

- **All these liabilities gives an insight how the liability can vary because of trend changes in the past.**
  - **By calculating the standard deviation**
- **This we use as an estimation of the possible impact of future trend changes**.

Uncertainty Trend

- **The *n* several trends are a sample of possible trends**
- **To calculate the ECtrend we use the Student t distribution (n-1 degrees of freedom)**

Uncertainty Trend *Example*

		single premium 4%				
		Term	**Pure end.**	**endowment**	**Annuity**	
Trend						
BE		0,0652	0,4084	0,4736	11,259	
from	**to**					
1949	1954	0,0961	0,3835	0,4797	11,265	
1954	1959	0,1010	0,3798	0,4809	11,351	
1959	1964	0,1308	0,3563	0,4871	10,541	
1964	1969	0,1020	0,3799	0,4819	10,688	
1969	1974	0,0823	0,3955	0,4778	10,810	
1974	1979	0,0650	0,4086	0,4736	11,837	
1979	1984	0,0700	0,4046	0,4747	11,109	
1984	1989	0,0620	0,4108	0,4728	11,501	
1989	1994	0,0623	0,4113	0,4736	11,409	
Stand. Div.		0,0222	0,0177	0,0045	0,397	
Stand. Div.		34,0%	4,3%	1,0%	3,5%	BE
MVM		47,7%	6,1%	1,3%	4,9%	BE
EC		162,7%	20,7%	4,6%	16,8%	BE

Uncertainty goal table

- **The uncertainty of the goal table can be based on expert opinion (shock because of medical development)**
- **By using uncertainty out of other prognosis**
- **In case the goal table is based on long term trends the same type of model can be used to as showed earlier, but now calculation uncertainty around the e_x in the goal year and based on long term historical trends**

Total uncertainty

- **The total uncertainty follows the sum of the uncertainty in the local trend and the uncertainty in the goal table**.

Total uncertainty

- **The total uncertainty follows the sum of the uncertainty in the local trend and the uncertainty in the goal table.**

% BE Liability

	no projection	incl. projection BE liability	delta	uncertainty trend 99.50%	uncertainty goal table 99.50%	total uncertainty 99.50%
Pure endowment age 25/65	0.1785	0.1918	7.4%	3.3%	2.3%	5.6%
immediate annuity age 65	11.1282	11.5121	3.5%	2.8%	2.4%	5.2%
deferred annuity age 25/65	1.9867	2.5324	27.5%	4.4%	24.1%	28.5%

Liability at 4% discount rate

Prognosis over 50 years

Trend observations 1947-2005 NL

Uncertainty goal table based on CBS information and medical expert ideas

Workshop

- Next time (Friday) we will try TOGETHER to develop a mortality prognosis table for Greece.

- No promises that it will work and we have one next weekend….